DIFFERENTIAL EQUATION

(10 MARKS)

(1% DIFFERENTIAL EQUATION)

(99% INTEGRATION)

Q: $(x^2+1)y\dfrac{dy}{dx} = x$

$y \xrightarrow{\text{diff}} \dfrac{dy}{dx}$

$\dfrac{dy}{dx} \xrightarrow{\text{integrate}} y$

$\int y \, dy = \frac{1}{2}\int \dfrac{2x}{\boxed{x^2+1}} \, dx$

(seperation of variables.)

$\square = x^2+1$

$\square' = 2x$

$\dfrac{y^2}{2} = \dfrac{1}{2}\ln(x^2+1) + c$

7 Given that $y = 0$ when $x = 1$, solve the differential equation

$$xy\frac{dy}{dx} = y^2 + 4,$$

obtaining an expression for y^2 in terms of x. [6]

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$xy \, dy = (y^2+4) \, dx$

$\frac{1}{2}\int \dfrac{2y}{\boxed{y^2+4}} \, dy = \int \dfrac{1}{x} \, dx$

DIFF EQ.

$\square = y^2+4$

$\square' = 2y$

$$\boxed{\frac{1}{2}\ln(y^2+4) = \ln x + c}$$

$y = 0$

$x = 1$

INTEG

$\frac{1}{2}\ln(0^2+4) = \ln 1 + c$

$$\boxed{\frac{1}{2}\ln 4 = c}$$

$$\frac{1}{2}\ln(y^2+4) = \ln x + \frac{1}{2}\ln 4$$

$$\frac{1}{2}\left[\ln(y^2+4) - \ln 4\right] = \ln x$$

$$\ln\left(\frac{y^2+4}{4}\right) = 2\ln x$$

$$\cancel{\ln}\left(\frac{y^2+4}{4}\right) = \cancel{\ln}\, x^2$$

$$\frac{y^2+4}{4} = x^2$$

$$y^2+4 = 4x^2$$

$$\boxed{y^2 = 4x^2 - 4}$$

SIMPLIFY (LOGS)

12 The variables x and y are related by the differential equation

$$\frac{dy}{dx} = \frac{6xe^{3x}}{y^2}.$$

It is given that $y = 2$ when $x = 0$. Solve the differential equation and hence find the value of y when $x = 0.5$, giving your answer correct to 2 decimal places. [8]

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$$\int y^2\,dy = \int 6xe^{3x}\,dx$$

$$\frac{y^3}{3} = 6\int \underbrace{x}_{u}\,\underbrace{e^{3x}}_{v}\,dx$$

Diff Equations.

DIFF OF U

INTEG OF V

$x \to \boxed{1}$

$\frac{1}{3}\int 3e^{\boxed{3x}}\,dx$

$\square = 3x$

$\square' = 3$

$$\boxed{\frac{e^{3x}}{3}}$$

$$u\int v\,dx - \int\left[\frac{du}{dx}\times\int v\,dx\right]dx$$

$$(x)\left(\frac{e^{3x}}{3}\right) - \int\left[1\times\frac{e^{3x}}{3}\right]dx$$

$$\frac{xe^{3x}}{3} - \frac{1}{3}\int e^{3x}\,dx$$

$$\frac{xe^{3x}}{3} - \frac{1}{3}\left(\frac{e^{3x}}{3}\right)$$

INTEGRATION.

$$\boxed{\frac{y^3}{3} = 6\left(\frac{xe^{3x}}{3} - \frac{e^{3x}}{9}\right) + C}$$

$y = 2$

$x = 0$

$$\frac{2^3}{3} = 6\left(\frac{0e^{3(0)}}{3} - \frac{e^{3(0)}}{9}\right) + c$$

$$\frac{8}{3} = 6\left(-\frac{1}{9}\right) + c$$

$$\boxed{c = \frac{10}{3}}$$

$$\frac{y^3}{3} = 6\left(\frac{xe^{3x}}{3} - \frac{e^{3x}}{9}\right) + \frac{10}{3}$$

$$\boxed{x = 0.5}$$

$$\frac{y^3}{} = 6\left(0.5e^{3(0.5)} - \frac{e^{3(0.5)}}{}\right) + \frac{10}{}$$

SIMPLIFY.

$\frac{}{3} \quad \left(\frac{}{3} \quad 9 \right) \quad 3$

$y = \boxed{}$

14 The variables x and y are related by the differential equation

$$x\frac{dy}{dx} = 1 - y^2.$$

When $x = 2$, $y = 0$. Solve the differential equation, obtaining an expression for y in terms of x. [8]

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DIFF EQ.

$$x\,dy = (1-y^2)\,dx$$

$$\int \frac{1}{\boxed{1-y^2}}\,dy = \int \frac{1}{x}\,dx$$

$\square = 1-y^2$

$\square' = -2y$

(M1) Partial fraction.

NOTE: JAB BHI DENOMINATOR KI DIFF IS NOT PRESENT IN NUMERATOR, 100% PARTIAL FRACTION.

$$\int \left(\frac{1}{2(1+y)} + \frac{1}{2(1-y)} \right) dy = \int \frac{1}{x}\,dx$$

$$\frac{1}{2}\left[\int \frac{1}{1+y}\,dy + (-1)\int \frac{-1}{\boxed{1-y}}\,dy \right] = \int \frac{1}{x}\,dx$$

$$\boxed{\frac{1}{2}\left[\ln(1+y) - \ln(1-y) \right] = \ln x + c}$$

$x=2$
$y=0$

ATION

INTEGR

$$\frac{1}{2}\left[\ln\left(\frac{1+y}{1-y}\right)\right] = \ln x + c$$

$$\frac{1}{2}\ln\left[\frac{1+0}{1-0}\right] = \ln 2 + c$$

$$\boxed{c = -\ln 2}$$

SIMPLIFY (LOGS)

$$\frac{1}{2}\ln\left(\frac{1+y}{1-y}\right) = \ln x - \ln 2$$

$$\frac{1}{2}\ln\left(\frac{1+y}{1-y}\right) = \ln\left(\frac{x}{2}\right)$$

$$\ln\left(\frac{1+y}{1-y}\right) = 2\ln\left(\frac{x}{2}\right)$$

$$\cancel{\ln}\left(\frac{1+y}{1-y}\right) = \cancel{\ln}\left(\frac{x}{2}\right)^2$$

$$\frac{1+y}{1-y} = \frac{x^2}{4}$$

$$4+4y = x^2 - x^2y$$

$$4y + x^2y = x^2 - 4$$

$$y(4+x^2) = x^2 - 4$$

$$y = \frac{x^2-4}{x^2+4}$$

(W1) PARTIAL FRACTION:

Partial Fractions.

$$\frac{1}{1-y^2}$$

$$\frac{1}{(1+y)(1-y)} \equiv \frac{A}{1+y} + \frac{B}{1-y} = \frac{1}{2(1+y)} + \frac{1}{2(1-y)}$$

$$\frac{1}{(1+y)(1-y)} = \frac{A(1-y)+B(1+y)}{(1+y)(1-y)}$$

$$\boxed{1 \equiv A(1-y)+B(1+y)}$$

$1-y=0$

$y=1$

$1 = B(1+1)$

$B = \frac{1}{2}$

$1+y=0$

$y=-1$

$1 = A(1-(-1))$

$A = \frac{1}{2}$

15 The variables x and y are related by the differential equation

$$(x^2+4)\frac{dy}{dx} = 6xy.$$

It is given that $y = 32$ when $x = 0$. Find an expression for y in terms of x. [6]

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DiFF EQ

$(x^2+4)\,dy = 6xy\,dx$

$\int \frac{1}{y}\,dy = \int \frac{6x}{x^2+4}\,dx$

$\int \frac{1}{y}\,dy = \frac{6}{2}\int \frac{2x}{\boxed{x^2+4}}\,dx$

$\square = x^2+4$

$\square' = 2x$

Integr.

$$\boxed{\ln y = 3\ln(x^2+4) + c}$$

$y=32$

$x=0$

$\ln 32 = 3\ln(0^2+4) + c$

$\ln 32 = 3\ln 4 + c$

$\ln 32 = \ln 4^3 + c$

$\ln 32 = \ln 64 + c$

$\ln 32 - \ln 64 = c$

$\ln\left(\frac{32}{64}\right) = c$

$c = \ln\left(\frac{1}{2}\right) = \ln 2^{-1} = \boxed{-\ln 2}$

Simplify LOGS-

$\ln y = 3\ln(x^2+4) - \ln 2$

$\ln y = \ln(x^2+4)^3 - \ln 2$

$$\ln y = \ln\left(\frac{(x^2+4)^3}{2}\right)$$

$$y = \frac{(x^2+4)^3}{2}$$

19 The variables x and y are related by the differential equation

$$\frac{dy}{dx} = \frac{6ye^{3x}}{2+e^{3x}}.$$

Given that $y = 36$ when $x = 0$, find an expression for y in terms of x. [6]

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20 The variables x and θ satisfy the differential equation

$$2\cos^2\theta \frac{dx}{d\theta} = \sqrt{(2x+1)},$$

and $x = 0$ when $\theta = \frac{1}{4}\pi$. Solve the differential equation and obtain an expression for x in terms of θ. [7]

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<u>Monday</u> 4 – 5 M1
5 – 6 S1

P3 (2pm to 4pm)